**Problem 1**

After drinking alcoholic beverages, the ethanol in the blood stream is continuously removed by metabolism in the liver (converted into carbon dioxide and water). The alcohol concentrations in the liver (CLe) and in the blood stream (CBe) are both functions of time, i.e. constantly changing. (See diagram below)

For simplicity, assume the rate of reaction in the liver is only dependent on the alcohol concentration in the liver. The rate of reaction is given as dCLe(t)/dt = - k\*CLe(t), where the concentration is in gm/L. (Note: in reality, this is a series of enzymatic reactions, but it is given as a 1st order reaction to simplify your exam modeling.)

CBe(t)

Flowrate = F

Diagram

Blood stream

Volume = VB (assumed constant)

Ethanol concentration = CBe(t)

Liver

Volume = VL (assumed constant)

Ethanol concentration = CLe(t)

CLe(t)

Flowrate = F

Assume both the blood stream and liver can be modeled as lumped parameter systems and that the exiting streams have the same ethanol concentration as the systems they are leaving.

**Your task is to model the ethanol concentrations in the bloodstream and liver as functions of time.**

Dimensions of parameters: CBe(t), CLe(t) – mass/volume

F – volume/time (assumed constant)

k – 1/time

Data: VB = 5 L (assumed constant) VL = 0.4 L (assumed constant)

F = 0.1 L/min (assumed constant) k = 0.2 min-1 (assumed constant)

Initial conditions:

CBe(0) = 1.5 gm/L CLe(0) = 1.5 gm/L (highly inebriated!)

**Part A.**

Using the system diagram given, develop the differential equations modeling the ethanol concentrations in the blood stream and the liver, i.e. dCBe(t)/dt = … and dCLe(t)/dt = …

Be sure to check your equations for dimensional consistency.

**Part B**

Using MathCad (or Matlab if desired), solve for CBe(t) and CLe(t). Provide appropriate plots for CBe(t) and CLe(t) for 0 < t< 100 min.

**Part C.**

The blood alcohol level must drop to less than 0.75 gm/L (0.08 % by volume) to be considered sober enough to drive. Using your models, calculate how long (minutes) it takes to decrease the blood alcohol level to become legally safe to drive.

**Problem 2 (Initial value problem)**

Solve the following differential equation and provide an appropriate plot of the solution, y(t), over the range 0<t<100.



Alternatively written ODE

y’’(t)=A\*y’(t)\*sin(t/3) – B\*y(t) + C\*t +D\*t2

Initial conditions: 

Values of constants:









**Problem 3 (Boundary Value Problem)**

Solve the following differential equation and provide appropriate plots of the solution, y(t), dy/dx, and d2y/dx2 over the range 0<x<10.









Alternatively written ODE:

y’’’(x)=[20\*sqrt(1+y’’(x)2 + y’’(x)2]/(100-x)

Problem 1

B.





C. approx. 82 min



Problem 2

Problem 3

